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bisected at one point of intersection with the second parabola, and quadrisected by the other, smaller divisions being made by the other parabolas. The length of the double normal is $6\sqrt{6}\,p$. It is divided by the axis into parts which are as 1 to 5, and by the evolute into parts which are as 5 to 27 and 1 to 3.

(8) The line AB, joining the feet of the normals, passes through the focus (as is true for any pair of perpendicular normals), is $4\frac{1}{2}$ times p in length, is trisected at the focus, is parallel to DE (joining the other ends of the normals through P) and equal to $\frac{1}{3}DE$. Both AB and DE are bisected by the diameter through P and are of course parallel to the tangent at its extremity.

(9) DE is itself a normal to the parabola at D. If we draw the normal at E, then DE bisects the angle between the other two normals through E, the two to the right forming with the axis an isosceles triangle.

(10) The cusp and the points P and I are collinear, P being the point of bisection. The focus and the points P and J are collinear, P being the point of quadrisection, and the join being perpendicular to the normal at E.

(11) P is equidistant from B and the focus. The vertex, A, B and the middle point of BD lie on a circle whose center is on AE. The point I is equidistant from the cusp and B. The focus is equidistant from the intersections of the evolute with the parabola and H. A circle with BD as a diameter passes through the vertex. D and B are equidistant from J and $AB = JE$.

(12) The perpendicular bisector of DH is tangent to the second parabola where BD cuts it and the normal to the evolute at this point passes through J. The perpendicular to DE at H passes through B.

(13) The chord joining the vertex with the upper intersection of the parabola and evolute is perpendicular to BD and trisected by it.

(14) The third normal from any point on the second parabola is trisected by the axis.

(15) The areas between the normal chords AE and BD and the parabola are equal. Moreover the three areas between the chords BE, AB, and AD and the parabola are equal.

(16) From any point whatever (a, b) the sum of the X intercepts is $2(a+p)$, that is, twice the distance of the point from the directrix, and is constant for all points on a line perpendicular to the axis. Likewise the sum of the focal distances of the feet of the three normals is constant and equal to $(2a-p)$.

A SIMPLE ALGEBRAIC PARADOX.

By J. L. COOLIDGE, Harvard Universtiy.

Given two linear homogeneous complex equations

$$(1)\qquad \begin{aligned}(a+bi)(p+qi)+(c+di)(r+si)&=0,\\(a'+b'i)(p+qi)+(c'+d'i)(r+si)&=0.\end{aligned}$$

In order that these should be compatible it is necessary and sufficient that

$$\begin{vmatrix} (a+bi) & (c+di) \\ (a'+b'i) & (c'+d'i) \end{vmatrix} = 0.$$

This complex equation is equivalent to the *two* real equations

$$(ac' - a'c) = (bd' - b'd),$$
$$(ad' + bc') = (a'd + b'c).$$

Both of these must be fulfilled if (1) is to subsist.

On the other hand equations (1) are equivalent to

$$\begin{aligned} ap - bq + cr - ds &= 0, \\ bp + aq + dr + cs &= 0, \\ a'p - b'q + c'r - d's &= 0, \\ b'p + a'q + d'r + c's &= 0. \end{aligned}$$

These are compatible when, and only when, a *single* equation is satisfied, namely

$$\begin{vmatrix} a & -b & c & -d \\ b & a & d & c \\ a' & -b' & c' & -d' \\ b' & a' & d' & c' \end{vmatrix} = 0.$$

Which answer is right?

ON A PURELY PROJECTIVE BASIS FOR THE THEORY OF INVOLUTION.

By D. N. LEHMER.

The theory of involution is usually based on foundations which are more or less metrical. It is customary to derive the more important properties by means of circles, or else by the use of the theory of the anharmonic ratio, which itself is usually based on metrical notions of areas and trigonometric functions. These methods have their advantages, but to the purist it seems unfortunate that the general, projective properties of involution should not be derived in a non-metrical way, and that the metrical properties should not be obtained, as is usual, by the introduction of the elements at infinity.

A purely projective discussion of the subject may be based on the following well-known and easily derived theorem on the complete quadrangle:[1]

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[1] See Reye, *Geometry of Position*, Holgate's translation, page 36.